



Mathematics

HP COMPUTER CURRICULUM

Number Sets

TEACHERS ADVISOR

HEWLETT  PACKARD

Hewlett-Packard
Computer Curriculum Series

mathematics
TEACHER'S ADVISOR

number sets

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This material is designed to be used with any Hewlett-Packard system with the BASIC programming language such as the 9830A Educational basic, and the 2000 and 3000 series systems.

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INTRODUCTION

This Mathematics Set of the Hewlett-Packard Computer Curriculum Series consists of a set of a Student Lab Book and a corresponding Teacher's Advisor. It was designed to help meet the need for computer-oriented problems in mathematics, providing students an opportunity to use a computer as a problem solving tool or a modeling device within a particular subject matter area.

The materials are designed for flexible use as desired by the individual instructor. The material and exercises in this unit are intended as an "enrichment" experience in the field of number sets. The unit begins with a section on integers, with exercises involving common operations which are easily performed with a computer. The Euclidean Algorithm and the "Sieve of Eratosthenes" are presented as alternatives to exhaustive search routines for finding greatest common divisors and prime numbers, respectively. The second section deals with rational numbers, with exercises involving denumerability and forming rational numbers from given integer sets. The final section introduces irrational numbers; in particular, methods are derived for computing π and finding the r th root of a number. Obviously, the unit is not intended to be a complete study of number sets; instead, each discussion is self-contained, so the concepts can be studied independently and in any sequence. Thus, the material can be used to supplement and enrich your curriculum in any fashion you choose.

The mathematical concepts needed for each exercise are briefly reviewed, but you may want your students to study these in greater detail before attempting the exercises, especially if their background in number sets is limited. A list of possible references is included at the end of each section. These references will also provide additional problems for your better students.

The degree of difficulty of the material is dependent upon the amount of assistance which you choose to provide. With no assistance, the better student should be challenged. However, given a good deal of assistance, any second year algebra student should be able to work out the exercises with no great difficulty. The level of the material is determined by the assumption that students taking second year algebra will be quite capable as a group.

The Student Book provides text material and programming exercises for the students. There is a problem analysis, including a suggested approach and a macro flow chart, for each exercise. The Teacher's Advisor contains an example of a program to solve each exercise, micro flow charts, and a brief discussion of the important elements of the exercise. The micro flow charts should be given to the students only after they have made an attempt to solve the problem on their own. For best results, you should study all the solutions until you are certain you have a complete grasp of the general methods. This should be done before assigning the material to the class.

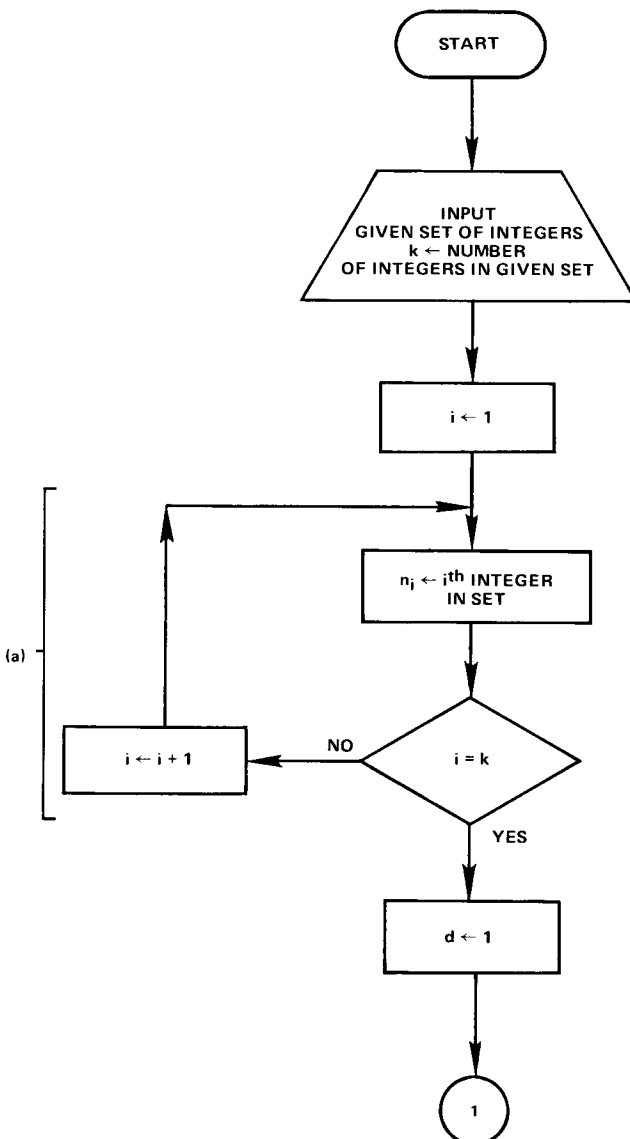
You will undoubtedly think of different programming methods or techniques as you study the example programs. Encourage the students to do the same. There are no *approved* solutions. All solutions are acceptable if they produce the correct results. At this level, there is no need for emphasis on the efficiency of a student's program. The important question is, does it work?

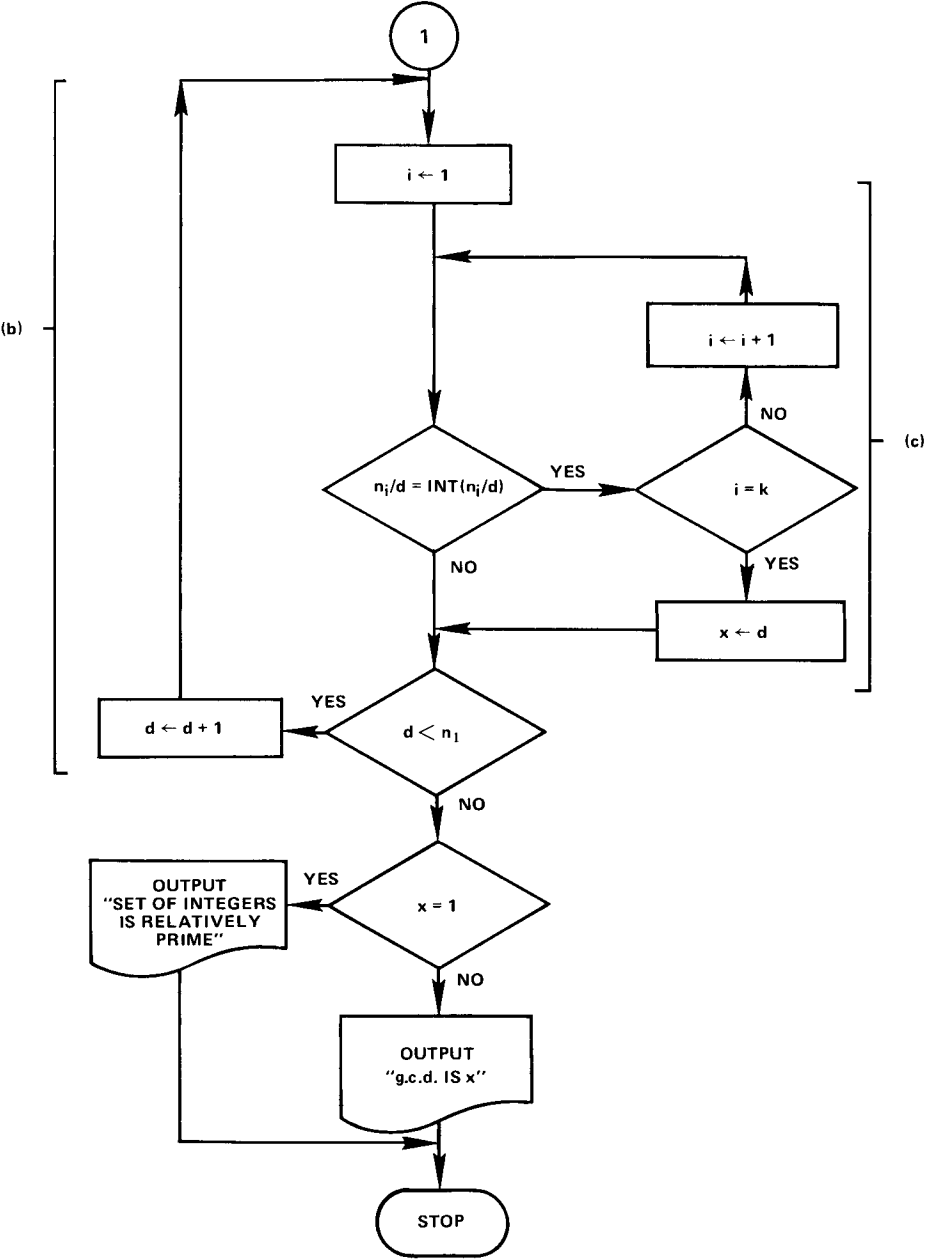
INTEGERS

EXERCISE 1 – Finding the Greatest Common Divisor

The loop indicated by (a) assigns subscripted variables to the integers in the set. The loop marked (b) selects the possible divisors of the set, the integers 1 through any number in the set. We pick the first number of the set as the upper limit of the possible divisors. If N_i/D is equal to the integer part of N_i/D , then we know that D is a divisor of N_i . The loop marked (c) determines if a possible divisor divides all integers in the set.

Micro Flow Chart





*Example Program**Exercise 1.*

```

10 REM--A PROGRAM TO DETERMINE IF INTERGERS OF A SET ARE RELATIVELY
20 REM--PRIME.
30 INPUT K
40 FOR I=1 TO K
50 READ N[I]
60 NEXT I
70 FOR D=1 TO N[I]
80 FOR I=1 TO K
90 IF N[I]/D#INT(N[I]/D) THEN 120
100 NEXT I
110 LET X=D
120 NEXT D
130 IF X=1 THEN 160
140 PRINT "G.C.D. IS";X
150 GOTO 180
160 PRINT "SET OF INTEGERS IS RELATIVELY PRIME."
170 DATA 91,26,169,286
180 END

```

RUN

```

?4
G.C.D.IS 13

```

DONE

```

170 DATA 71,251,149,353
RUN

```

```

?4
SET OF INTEGERS IS RELATIVELY PRIME.

```

DONE

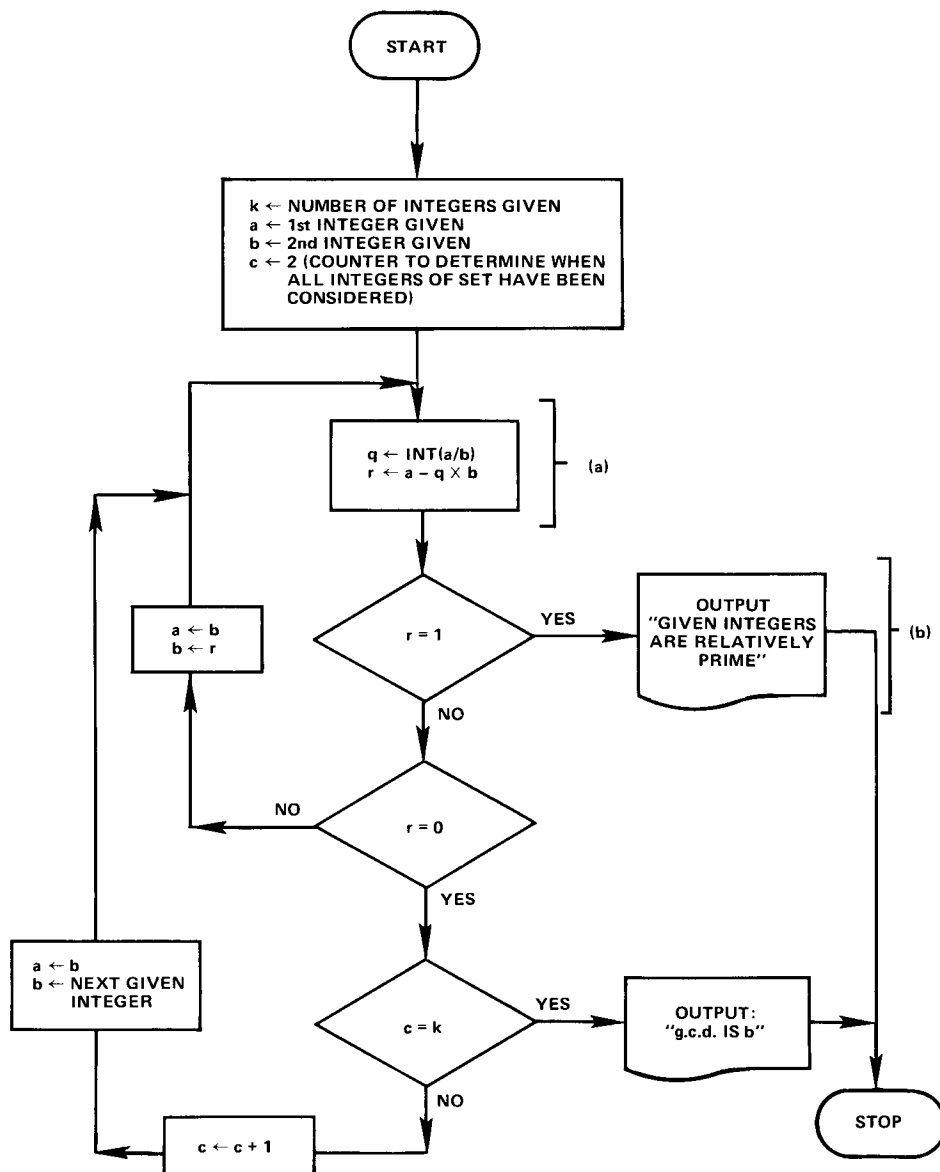
EXERCISE 2 – Using the Euclidean Algorithm

This problem is a good problem to demonstrate how the repetitive process of an algorithm is symbolized for programming purposes. The student should be able to write the program after careful study of the macro flow chart and the discussion in the Student Book. The student should recognize that the algorithm is applied in the following way:

- (1) $a \leftarrow$ first integer in given set, $b \leftarrow$ second integer in given set
- (2) Output g.c.d. of a and b
- (3) $a \leftarrow$ g.c.d. of a and b , $b \leftarrow$ next integer in given set
- (4) Go to (2)

The flow chart explains the procedure used. In the box labeled (a), the integral part of the quotient, q , for each a/b is determined and the remainder, r , is computed. Note that at (b) the entire set is labeled relatively prime if any two numbers being tested are found to be relatively prime.

Micro Flow Chart



*Example Program**Exercise 2.*

```
1  REM--A PROGRAM TO DETERMINE IF A SET OF ELEMENTS ARE RELATIVELY
2  REM--PRIME BY APPLYING THE EUCLIDEAN ALGORITHM.  ON THE DATA LINE
3  REM--ENTER THE NUMBER OF INTEGERS IN THE SET FOLLOWED BY THE INTEGERS
4  REM--IN THE SET
5  DATA 4,71,251,149,353
10  READ K,A,B
20  C=2
30  Q=INT(A/B)
40  R=A-Q*B
50  IF R=1 THEN 160
60  IF R=0 THEN 100
70  A=B
80  B=R
90  GOTO 30
100 IF C=K THEN 140
105 C=C+1
110 A=B
120 READ B
130 GOTO 30
140 PRINT "G.C.D.IS";B
150 GOTO 170
160 PRINT "GIVEN INTEGERS ARE RELATIVELY PRIME"
170 END
```

RUN

GIVEN INTEGERS ARE RELATIVELY PRIME

DONE

```
5 DATA 4,91,26,169,286
RUN
```

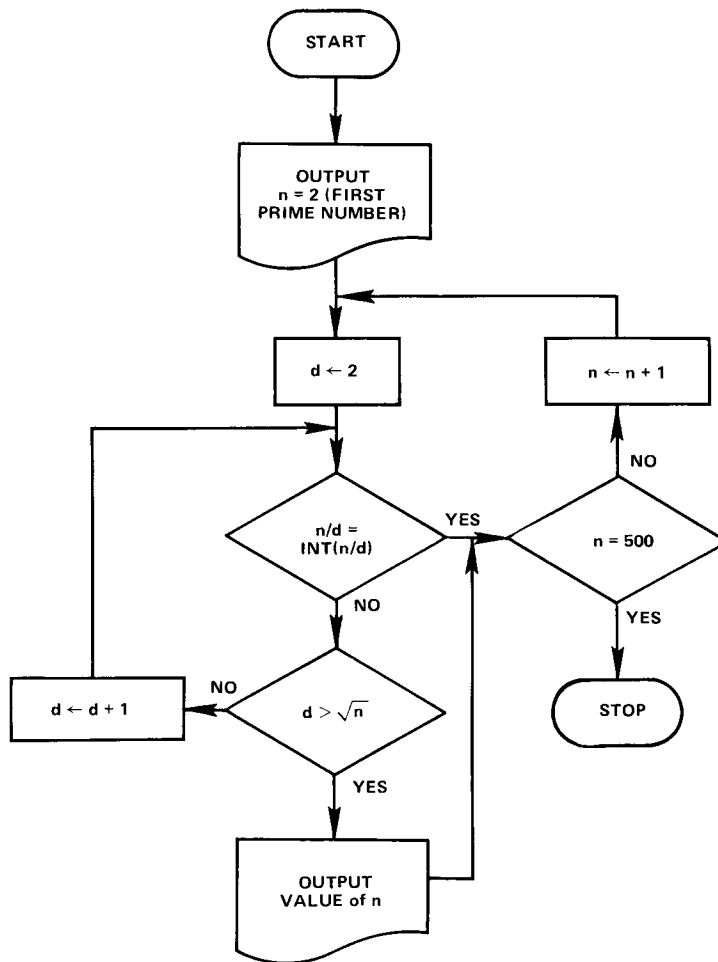
G.C.D.IS 13

DONE

EXERCISE 3 – Prime Numbers

Students should be able to complete this exercise without much assistance. Any questions will probably involve where to start their search and when to stop. We start with $n = 2$ because 2 is not considered a prime number. Our first divisor is 2, because every prime number has 1 as a divisor. We can stop when the divisor is greater than \sqrt{n} , since any divisor of n must also be a divisor of \sqrt{n} ($n = a^2 = a \times a$; therefore any divisor of n is necessarily a divisor of a).

Micro Flow Chart



Example Program

Exercise 3.

```

5  REM--A PROGRAM TO FIND ALL PRIME NUMBERS FROM 1 TO 500.
10  N=2
15  GOTO 70
20  D=2
30  IF N/D=INT(N/D) THEN 80
40  IF D>SQR(N) THEN 70
50  D=D+1
60  GOTO 30
70  PRINT N;
80  IF N=500 THEN 110
90  N=N+1
100 GOTO 20
110 END
  
```

RUN

2	3	5	7	11	13	17	19	23	29	31	37
41	43	47	53	59	61	67	71	73	79	83	89
97	101	103	107	109	113	127	131	137	139	149	151
157	163	167	173	179	181	191	193	197	199	211	223
227	229	233	239	241	251	257	263	269	271	277	281
283	293	307	311	313	317	331	337	347	349	353	359
367	373	379	383	389	397	401	409	419	421	431	433
439	443	449	457	461	463	467	479	487	491	499	

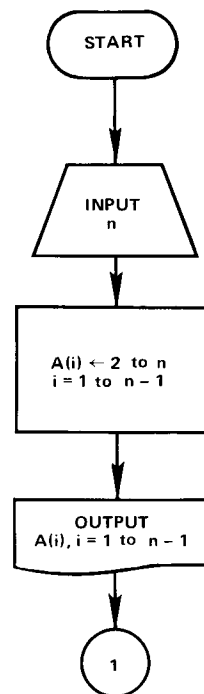
DONE

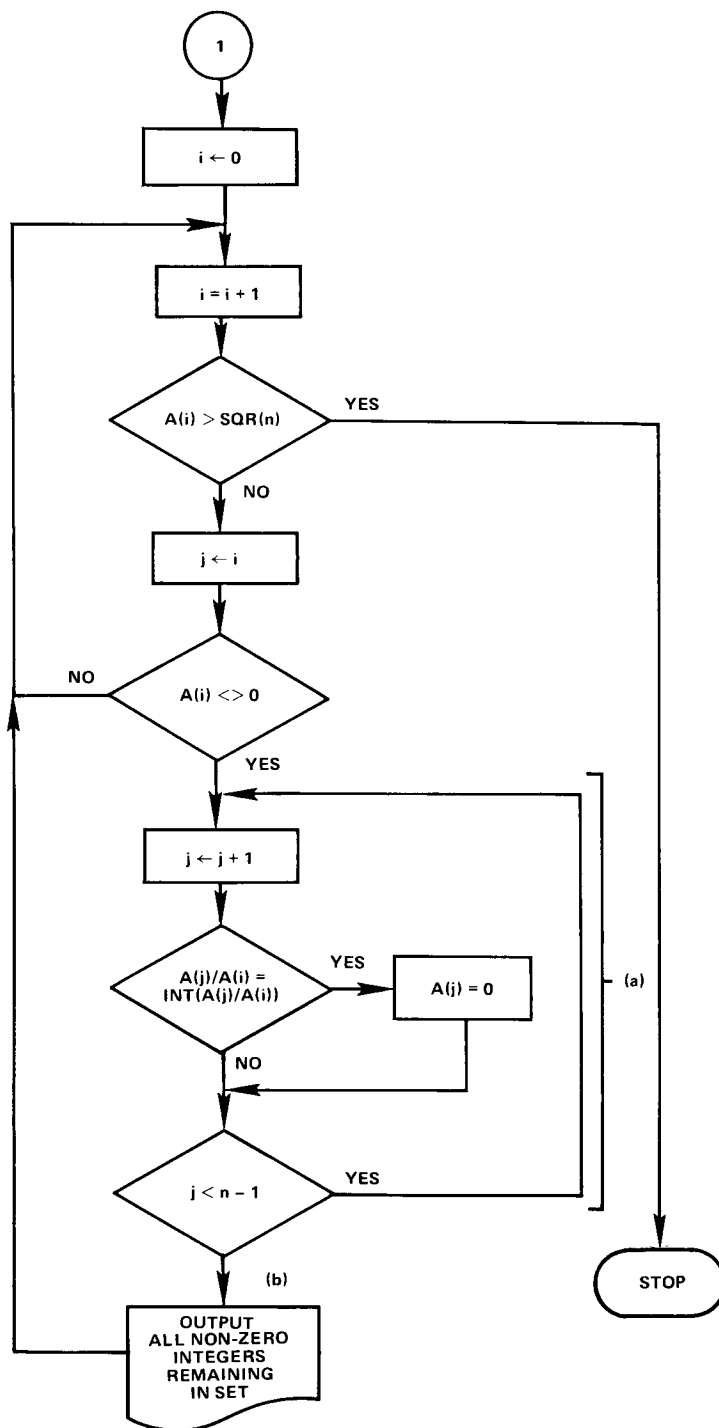
EXERCISE 4 – Determining Prime Numbers by “Sieve of Eratosthenes” Method

This problem requires the student to abstract an algorithm from the simulation of a given process. Make sure that your students actually use the “Sieve of Eratosthenes” method, i.e., eliminating multiples of each successive prime from 2 to \sqrt{n} , where n is the last integer in the set.

The flow chart below illustrates the correct application of this method. The loop marked (a) determines all multiples of a particular prime and sets them equal to zero. Each time the set is printed, elements equal to zero are omitted.

Micro Flow Chart





*Example Program**Exercise 4.*

```

10 REM--A PROGRAM MODELING THE SIEVE OF ERATOSTHENES. INPUT THE
20 REM--LARGEST NUMBER IN THE SIEVE.
30 INPUT N
40 DIM A(200)
50 FOR I=1 TO N-1
60 LET A(I)=I+1
70 PRINT A(I);
80 NEXT I
90 PRINT
100 PRINT
110 LET I=0
120 LET I=I+1
125 IF A(I)>SQR(N) THEN 280
130 LET J=I
140 IF A(I) <> 0 THEN 160
150 GOTO 120
160 J=J+1
170 IF A(J)/A(I)=INT(A(J)/A(I)) THEN 190
180 GOTO 200
190 A(J)=0
200 IF J<N-1 THEN 160
210 PRINT
220 PRINT
230 FOR K=1 TO N-1
240 IF A(K)=0 THEN 260
250 PRINT A(K);
260 NEXT K
270 GOTO 120
280 END

```

RUN

```

?30
 2   3   4   5   6   7   8   9  10  11  12  13
14  15  16  17  18  19  20  21  22  23  24  25
26  27  28  29  30

 2   3   5   7   9  11  13  15  17  19  21  23
25  27  29

 2   3   5   7  11  13  17  19  23  25  29

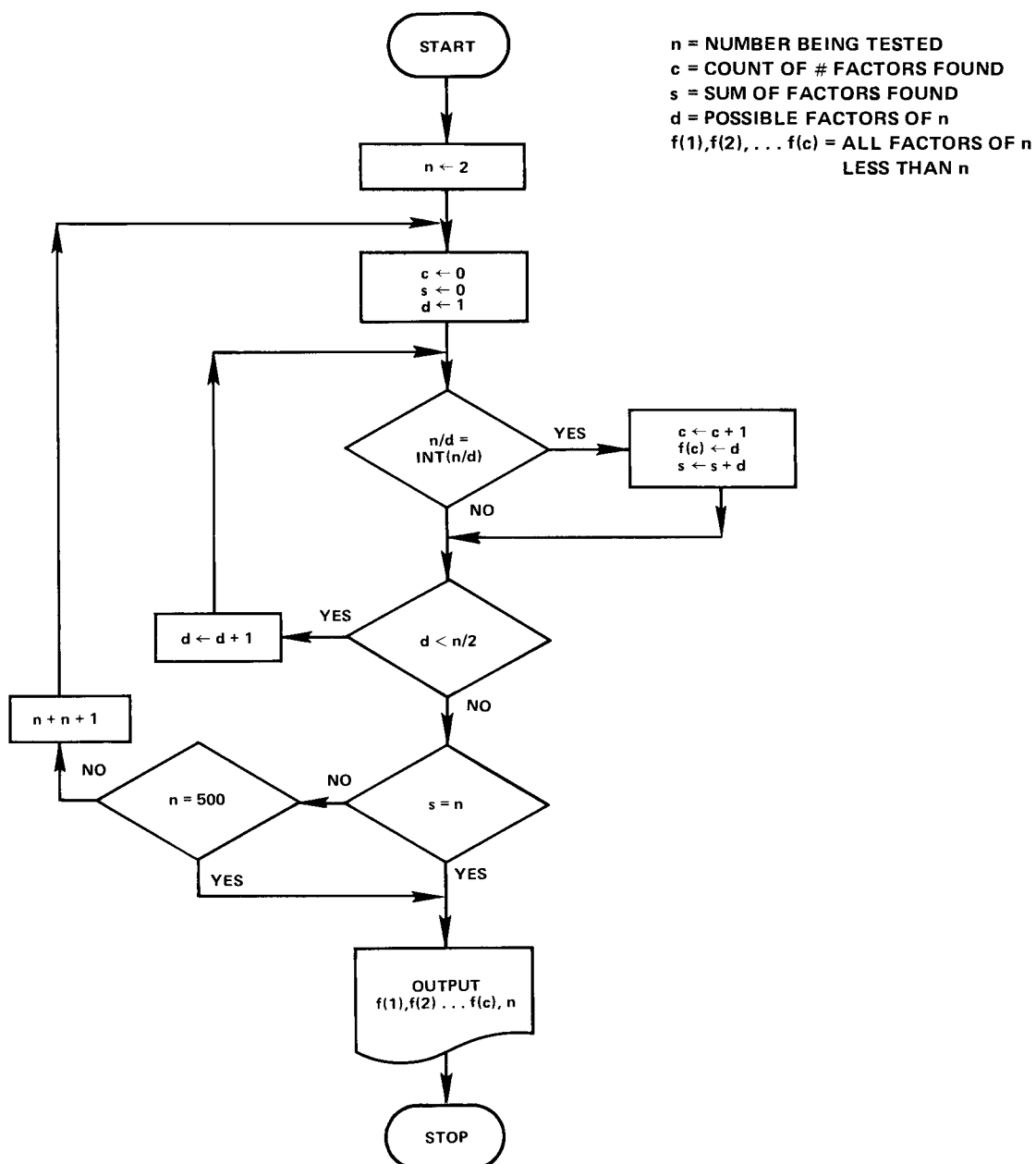
 2   3   5   7  11  13  17  19  23  29
DONE

```

EXERCISE 5 – Determining Perfect Numbers

This problem introduces the student to interesting related concepts, i.e., Mersene Primes and Amicable Numbers. Hopefully, the student will be inspired to investigate some of the ideas he will come across when he consults the suggested references.

Micro Flow Chart



*Example Program**Exercise 5.*

```

5  REM--A PROGRAM TO FIND ALL PERFECT NUMBERS LESS THAN OR EQUAL TO 500.
10  DIM F[50]
20  FOR N=2 TO 500
30  LET C=0
40  LET S=0
50  FOR D=1 TO N/2
60  IF N/D=INT(N/D) THEN 110
70  NEXT D
80  GOTO 150
90  NEXT N
100 GOTO 230
110 LET C=C+1
120 LET F[C]=D
130 LET S=S+D
140 GOTO 70
150 IF S=N THEN 170
160 GOTO 90
170 FOR I=1 TO C
180 PRINT F[I];", ";
190 NEXT I
200 PRINT "ARE FACTORS OF THE PERFECT NUMBER"N
210 PRINT
220 GOTO 90
230 END

```

RUN

```

1    , 2    , 3    ,ARE FACTORS OF THE PERFECT NUMBER 6
1    , 2    , 4    , 7    , 14    ,ARE FACTORS OF THE PERFECT NUMBER 28
1    , 2    , 4    , 8    , 16    , 31    , 62    , 124    , 248    ,
ARE FACTORS OF THE PERFECT NUMBER 496

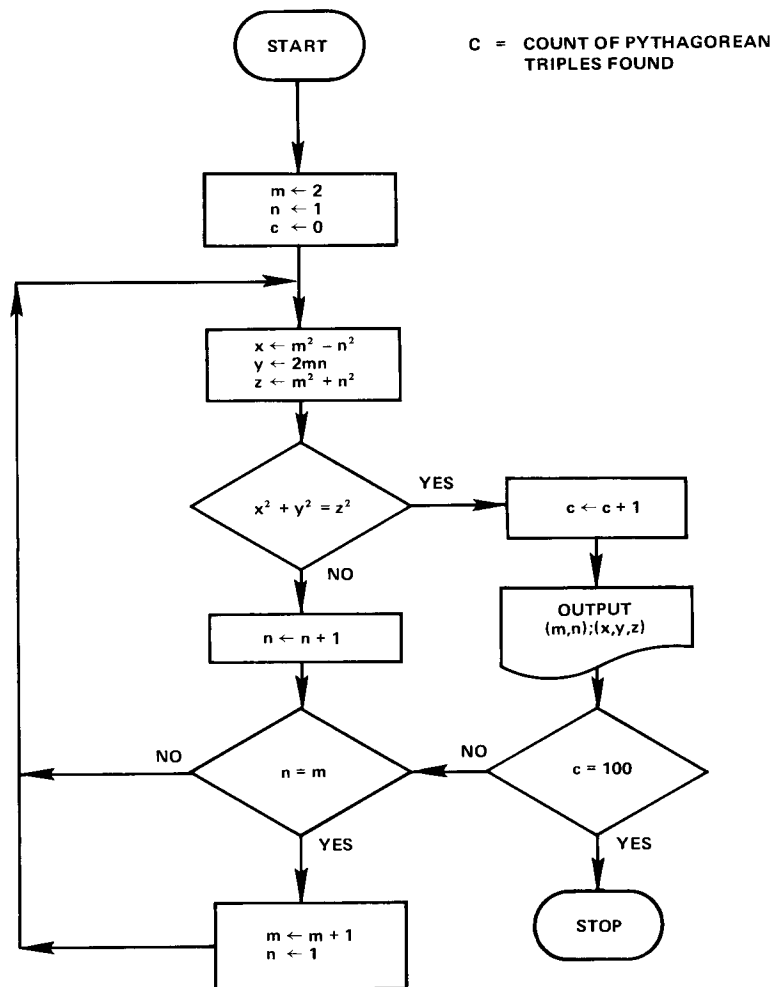
```

DONE

EXERCISE 6 – Finding Pythagorean Triples

Have students research why Pythagorean triples (x,y,z) must meet the relationships $x = m^2 - n^2$, $y = 2mn$, and $z = m^2 + n^2$, m and n positive integers, $m > n$.

Micro Flow Chart



*Example Program**Exercise 6(a).*

```

5  REM--A PROGRAM TO FIND PYTHAGOREAN TRIPLES.
10  M=2
20  N=1
30  C=0
40  X=M2-N2
50  Y=2*M*N
60  Z=M2+N2
70  IF X2+Y2=Z2 THEN 140
80  LET N=N+1
90  IF N=M THEN 110
100 GOTO 40
110 LET M=M+1
120 LET N=1
125 IF N=M THEN 90
130 GOTO 40
140 C=C+1
150 PRINT "(",M,"",",",N,"")""("X,"",",Y,"",",",Z,"")"
160 IF C=100 THEN 180
170 GOTO 80
180 END

```

RUN

```

( 2      , 1      )( 3      , 4      , 5      )
( 3      , 1      )( 8      , 6      , 10     )
( 3      , 2      )( 5      , 12     , 13     )
( 4      , 1      )( 15     , 8      , 17     )
( 4      , 2      )( 12     , 16     , 20     )
( 4      , 3      )( 7      , 24     , 25     )
( 5      , 1      )( 24     , 10     , 26     )
( 5      , 2      )( 21     , 20     , 29     )
( 5      , 3      )( 16     , 30     , 34     )
( 5      , 4      )( 9      , 40     , 41     )
( 6      , 1      )( 35     , 12     , 37     )
( 6      , 2      )( 32     , 24     , 40     )
( 6      , 3      )( 27     , 36     , 45     )
( 6      , 4      )( 20     , 48     , 52     )
( 6      , 5      )( 11     , 60     , 61     )
( 7      , 1      )( 48     , 14     , 50     )
( 7      , 2      )( 45     , 28     , 53     )
( 7      , 3      )( 40     , 42     , 58     )
( 7      , 4      )( 33     , 56     , 65     )
( 7      , 5      )( 24     , 70     , 74     )
( 7      , 6      )( 13     , 84     , 85     )
( 8      , 1      )( 63     , 16     , 65     )
( 8      , 2      )( 60     , 32     , 68     )
( 8      , 3      )( 55     , 48     , 73     )
( 8      , 4      )( 48     , 64     , 80     )
( 8      , 5      )( 39     , 80     , 89     )
( 8      , 6      )( 28     , 96     , 100    )
( 8      , 7      )( 15     , 112    , 113    )
( 9      , 1      )( 80     , 18     , 82     )
( 9      , 2      )( 77     , 36     , 85     )
( 9      , 3      )( 72     , 54     , 90     )
( 9      , 4      )( 65     , 72     , 97     )
( 9      , 5      )( 56     , 90     ,
STOP

```

MATHEMATICS

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Example Program

Exercise 6(b).

```
10  REM--A PROGRAM TO FIND PRIMITIVE PYTHAGOREAN TRIPLES
20  M=2
30  N=1
40  C=0
50  X=M+2-N+2
60  Y=2*M*N
70  Z=M+2+N+2
80  IF X+2+Y+2=Z+2 THEN 160
90  LET N=N+1
100 IF N=M THEN 120
110 GOTO 50
120 LET M=M+1
130 LET N=1
140 GOTO 50
160 LET A=X
170 LET B=Y
180 Q=INT(A/B)
190 R=A-Q*B
200 IF R=1 THEN 250
210 IF R=0 THEN 90
220 A=B
230 B=R
240 GOTO 180
250 C=C+1
260 PRINT "(",M,";",N,")""(",X,";",Y,";",Z,")"
270 IF C=100 THEN 290
280 GOTO 90
290 END
```

RUN

```
( 2 , 1 )( 3 , 4 , 5 )
( 3 , 2 )( 5 , 12 , 13 )
( 4 , 1 )( 15 , 8 , 17 )
( 4 , 3 )( 7 , 24 , 25 )
( 5 , 2 )( 21 , 20 , 29 )
( 5 , 4 )( 9 , 40 , 41 )
( 6 , 1 )( 35 , 12 , 37 )
( 6 , 5 )( 11 , 60 , 61 )
( 7 , 2 )( 45 , 28 , 53 )
( 7 , 4 )( 33 , 56 , 65 )
( 7 , 6 )( 13 , 84 , 85 )
( 8 , 1 )( 63 , 16 , 65 )
( 8 , 3 )( 55 , 48 , 73 )
( 8 , 5 )( 39 , 80 , 89 )
( 8 , 7 )( 15 , 112 , 113 )
( 9 , 2
STOP
```

RATIONAL NUMBERS

EXERCISE 7 – Denumeration of the Set of Positive Rational Numbers

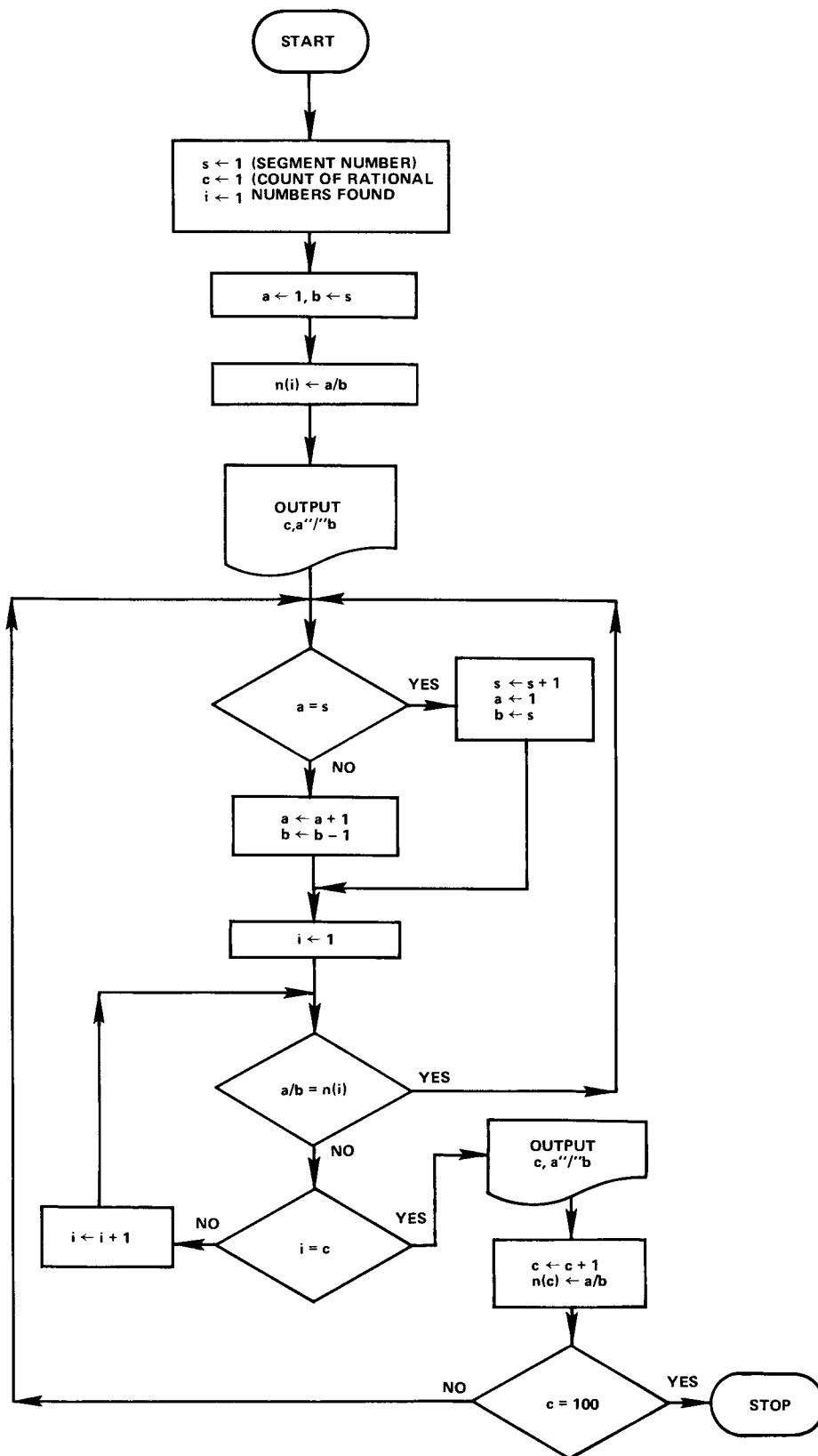
This exercise is another which requires that the student formulate an algorithm from the output pattern generated by hand.

By reordering the terms in alternating segments (diagonals) of the sequence, we have a new sequence in which the terms in every segment are in increasing order:

$$\begin{array}{l} 1/1 \mid 1/2, 2/1 \mid 1/3, 2/2, 3/1 \mid 1/4, 2/3, 3/2, 4/1 \mid \\ \mid 1/5, 2/4, 3/3, 4/2, 5/1 \mid 1/6, 2/5, 3/4, \dots \end{array}$$

If you look closer at this sequence, you see that the numerators within each segment are increasing, while the denominators are decreasing. In general, then, we see that in segment s , the numbers are $1/s, 2/(s-1), 3/(s-2) \dots (s-2)/3, (s-1)/2, s/1$.

In the flow chart, note that $a/$ ' b represents the value of a "over" the value of b , and a/b represents the quotient of a divided by b .



*Example Program**Exercise 7(a).*

```

1  DIM N(100)
5  REM--A PROGRAM TO DEMONSTRATE THE DENUMERABILITY OF THE SET
6  REM-- RATIONAL NUMBERS.
10 S=1
20 I=1
30 C=1
40 A=1
50 B=S
55 N(I)=A/B
60 PRINT C;A"/"B
70 IF A=S THEN 190
80 A=A+1
90 B=B-1
100 I=1
110 IF A/B=N(I) THEN 70
120 IF I=C THEN 150
130 I=I+1
140 GOTO 110
150 C=C+1
160 PRINT C;A"/"B
165 LET N(C)=A/B
170 IF C=100 THEN 230
180 GOTO 70
190 S=S+1
200 A=1
210 B=S
220 GOTO 100
230 END

```

RUN

```

1      1      / 1
2      1      / 2
3      2      / 1
4      1      / 3
5      3      / 1
6      1      / 4
7      2      / 3
8      3      / 2
9      4      / 1
10     1      / 5
11     5      / 1
12     1      / 6
13     2      / 5
14     3      / 4
15     4      / 3
16     5      / 2
17     6      / 1
18     1      / 7
19     3      / 5
20     5      / 3
21     7      / 1
22     1      / 8
23     2      / 7
24     4      / 5
25     5      / 4
26     7      / 2
27     8      / 1
28     1      / 9
29     3      / 7
30     7      / 3
31     9      / 1
32     1      / 10
33     2      / 9
34     3      / 8

```

STOP

Exercise 7(b).

```

10 DIM N[100]
20 I=1
30 REM--A PROGRAM TO DEMONSTRATE THE DENUMERABILITY OF THE SET
40 REM-- RATIONAL NUMBERS.
50 PRINT 1;0"/"1
60 N[I]=0/1
70 S=1
80 I=1
90 C=2
100 A=1
110 B=S
120 N[C]=A/B
130 PRINT C;A"/"B
140 I=I+1
150 PRINT C;"-"A"/"B
160 IF A=S THEN 320
170 A=A+1
180 B=B-1
190 I=1
200 IF A/B=N[I] THEN 160
210 IF I=C THEN 240
220 I=I+1
230 GOTO 200
240 C=C+1
250 PRINT C;A"/"B
260 LET N[C]=A/B
270 C=C+1
280 PRINT C;"-"A"/"B
290 N[C]=A/B
300 IF C=100 THEN 360
310 GOTO 160
320 S=S+1
330 A=1
340 B=S
350 GOTO 190
360 END

```

RUN

```

1      0      / 1
2      1      / 1
2     - 1      / 1
3      1      / 2
4     - 1      / 2
5      2      / 1
6     - 2      / 1
7      1      / 3
8     - 1      / 3
9      3      / 1
10     - 3      / 1
11     1      / 4
12     - 1      / 4
13     2      / 3
14     - 2      / 3
15     3      / 2
16     - 3      / 2
17     4      / 1
18     - 4      / 1
STOP

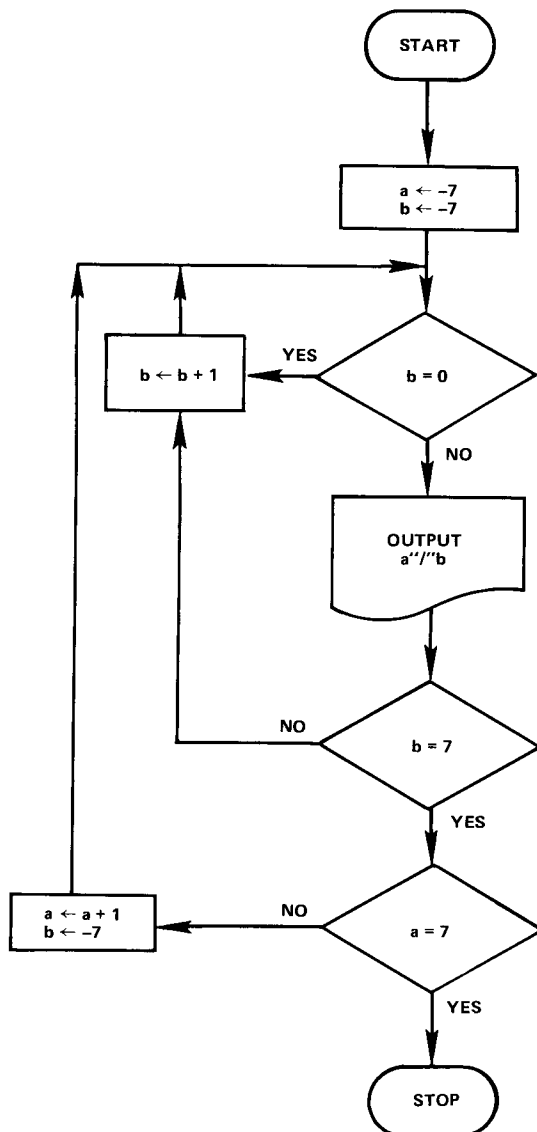
```

EXERCISE 8 – Forming Rational Numbers, Using All Integers in a Given Real Number Interval

This program is significant because it models the definition of a rational number. The student will have to deal with the condition that division by zero is not possible.

Micro Flow Chart

Part (a)



MATHEMATICS

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Example Program

Exercise 8(a).

```

5  REM--A PROGRAM TO FORM ALL POSSIBLE RATIONAL NUMBERS
6  REM--USING THE INTEGERS OVER A GIVEN REAL NUMBER INTERVAL.
10  FOR A=-7 TO 7
20  FOR B=-7 TO 7
30  IF B=0 THEN 50
40  PRINT A"/"B,
50  NEXT B
60  NEXT A
80  END

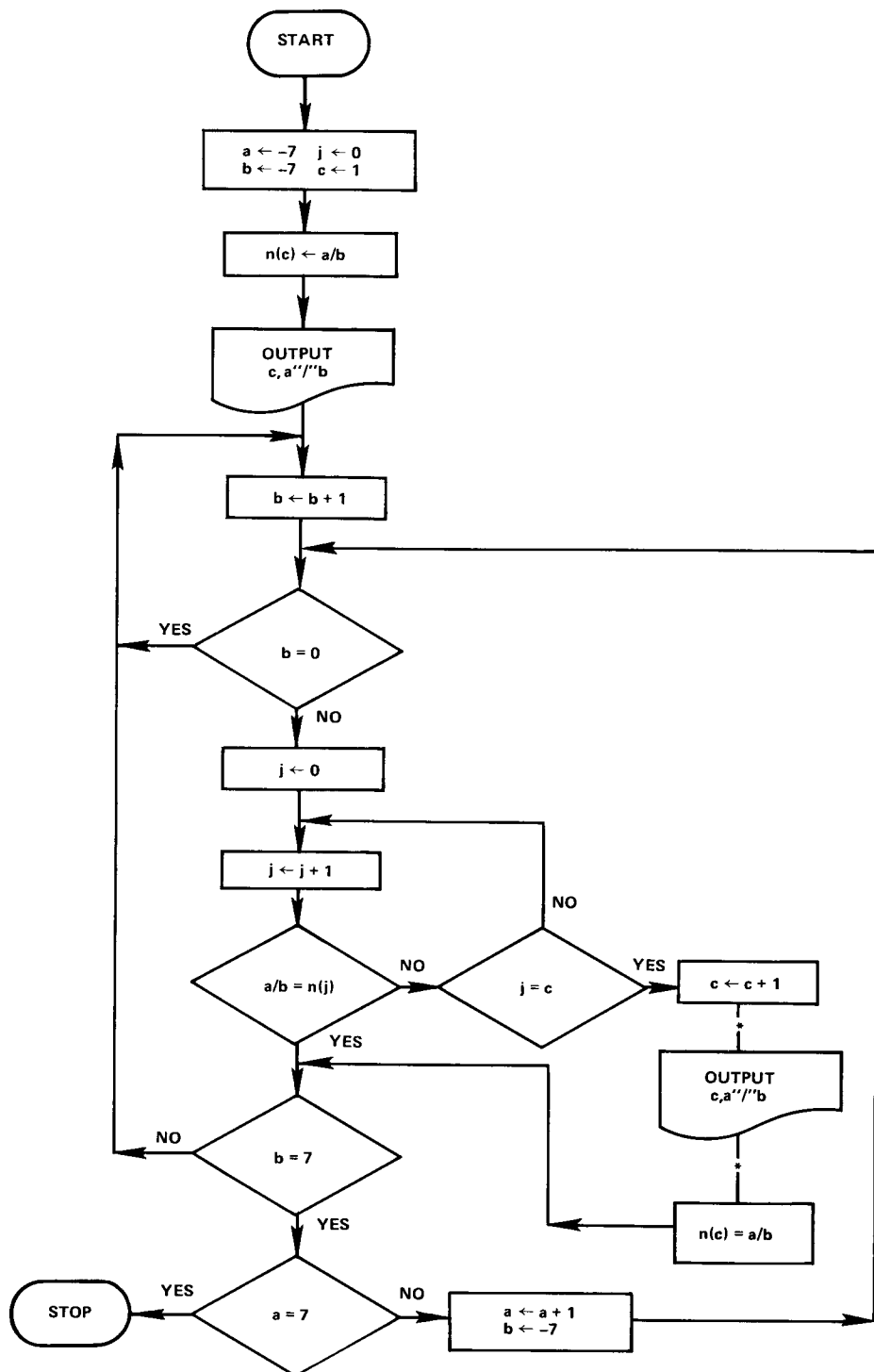
```

RUN

-7	/-7	-7	/-6	-7	/-5	-7	/-4	-7	/-3
-7	/-2	-7	/-1	-7	/ 1	-7	/ 2	-7	/ 3
-7	/ 4	-7	/ 5	-7	/ 6	-7	/ 7	-6	/-7
-6	/-6	-6	/-5	-6	/-4	-6	/-3	-6	/-2
-6	/-1	-6	/ 1	-6	/ 2	-6	/ 3	-6	/ 4
-6	/ 5	-6	/ 6	-6	/ 7	-5	/-7	-5	/-6
-5	/-5	-5	/-4	-5	/-3	-5	/-2	-5	/-1
-5	/ 1	-5	/ 2	-5	/ 3	-5	/ 4	-5	/ 5
-5	/ 6	-5	/ 7	-4	/-7	-4	/-6	-4	/-5
-4	/-4	-4	/-3	-4	/-2	-4	/-1	-4	/ 1
-4	/ 2	-4	/ 3	-4	/ 4	-4	/ 5	-4	/ 6
-4	/ 7	-3	/-7	-3	/-6	-3	/-5	-3	/-4
-3	/-3	-3	/-2	-3	/-1	-3	/ 1	-3	/ 2
-3	/ 3	-3	/ 4	-3	/ 5	-3	/ 6	-3	/ 7
-2	/-7	-2	/-6	-2	/-5	-2	/-4	-2	/-3
-2	/-2	-2	/-1	-2	/ 1	-2	/ 2	-2	/ 3
-2	/ 4	-2	/ 5	-2	/ 6	-2	/ 7	-1	/-7
-1	/-6	-1	/-5	-1	/-4	-1	/-3	-1	/-2
-1	/-1	-1	/ 1	-1	/ 2	-1	/ 3	-1	/ 4
-1	/ 5	-1	/ 6	-1	/ 7	0	/-7	0	/-6
0	/-5	0	/-4	0	/-3	0	/-2	0	/-1
0	/ 1	0	/ 2	0	/ 3	0	/ 4	0	/ 5
0	/ 6	0	/ 7	1	/-7	1	/-6	1	/-5
1	/-4	1	/-3	1	/-2	1	/-1	1	/ 1
1	/ 2	1	/ 3	1	/ 4	1	/ 5	1	/ 6

Micro Flow Chart

Part (b).



(*) For Part (c). A section of flow chart will replace this flow chart between the *.

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Example Program

Exercise 8(b)

```
5  REM-- A PROGRAM THAT WILL PRINT ALL MEANINGFUL RATIONAL
6  REM-- EXPRESSIONS THAT CAN BE FORMED FROM THE INTEGERS OVER
7  REM--A REAL NUMBER INTERVAL. EXPRESSIONS THAT HAVE THE SAME
9  REM--VALUE AS SOME EXPRESSION ALREADY PRINTED WILL NOT BE PRINTED.
10 READ A,B,C,J
15 DIM N(100)
20 DATA -7,-7,1,0
40 PRINT C;A"/"B
46 LET N(C)=A/B
50 LET B=B+1
60 IF B=0 THEN 50
65 J=0
70 LET J=J+1
80 IF A/B=N(J) THEN 110
90 IF J=C THEN 125
100 GOTO 70
110 IF B=7 THEN 170
120 GOTO 50
125 C=C+1
130 PRINT C;A"/"B
150 LET N(C)=A/B
160 GOTO 110
170 IF A=7 THEN 210
180 LET A=A+1
190 LET B=-7
200 GOTO 60
210 END
```

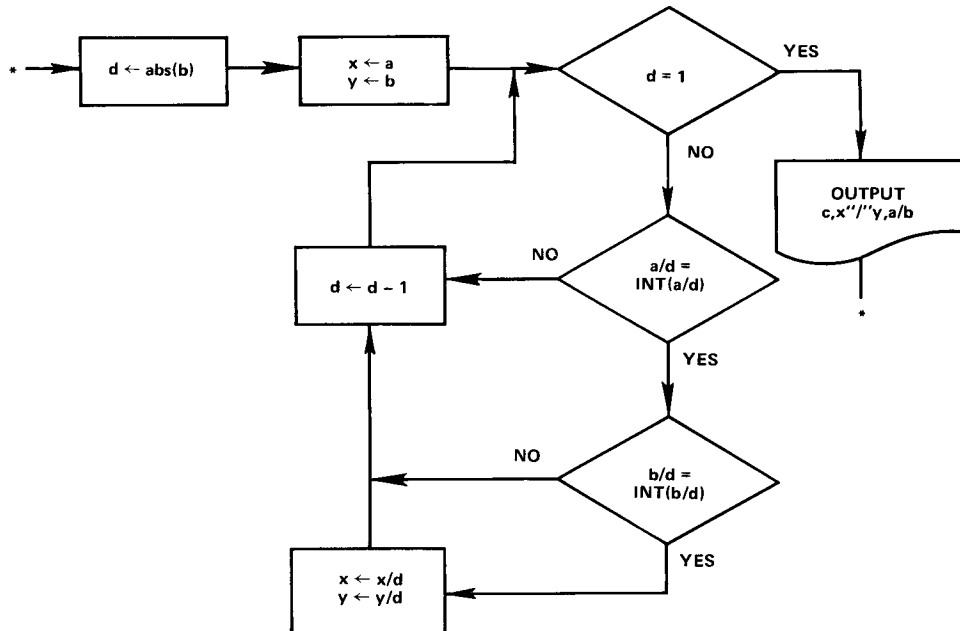
RUN

```
1  -7    /-7
2  -7    /-6
3  -7    /-5
4  -7    /-4
5  -7    /-3
6  -7    /-2
7  -7    /-1
8  -7    / 1
9  -7    / 2
10 -7    / 3
11 -7    / 4
12 -7    / 5
13 -7    / 6
14 -7    / 7
15 -6    /-7
16 -6    /-5
17 -6    /-4
18 -6    /-3
19 -6    /-2
STOP
```

Micro Flow Chart

Part (c).

Insert the following flow chart section as indicated in flow chart of Part (b).



Example Program

Exercise 8(c).

```

10 REM--A PROGRAM TO PRINT OUT IN REDUCED FORM AND DECIMAL FORM
20 REM--ALL RATIONAL NUMBERS THAT CAN BE FORMED FROM THE INTEGERS
30 REM --OVER SOME REAL NUMBER INTERVAL.
40 READ A,B,C,J
50 DIM N[100]
60 DATA -7,-7,1,0
70 PRINT C;A"/"B;A/B
80 LET N[C]=A/B
90 LET B=B+1
100 IF B=0 THEN 90
110 J=0
120 LET J=J+1
130 IF A/B=N[J] THEN 160
140 IF J=C THEN 180
150 GOTO 120
160 IF B=7 THEN 310
170 GOTO 90
180 C=C+1
190 LET D=ABS(B)
191 LET X=A
192 LET Y=B
195 IF D=1 THEN 280
200 IF A/D=INT(A/D) THEN 240
220 LET D=D-1
230 GOTO 195
240 IF B/D=INT(B/D) THEN 260
250 GOTO 220
260 LET X=X/D
270 LET Y=Y/D
275 GOTO 220
280 PRINT C;X"/"Y;A/B
290 LET N[C]=A/B
300 GOTO 160
310 IF A=7 THEN 350
320 LET A=A+1
330 LET B=-7
340 GOTO 100
350 END

```

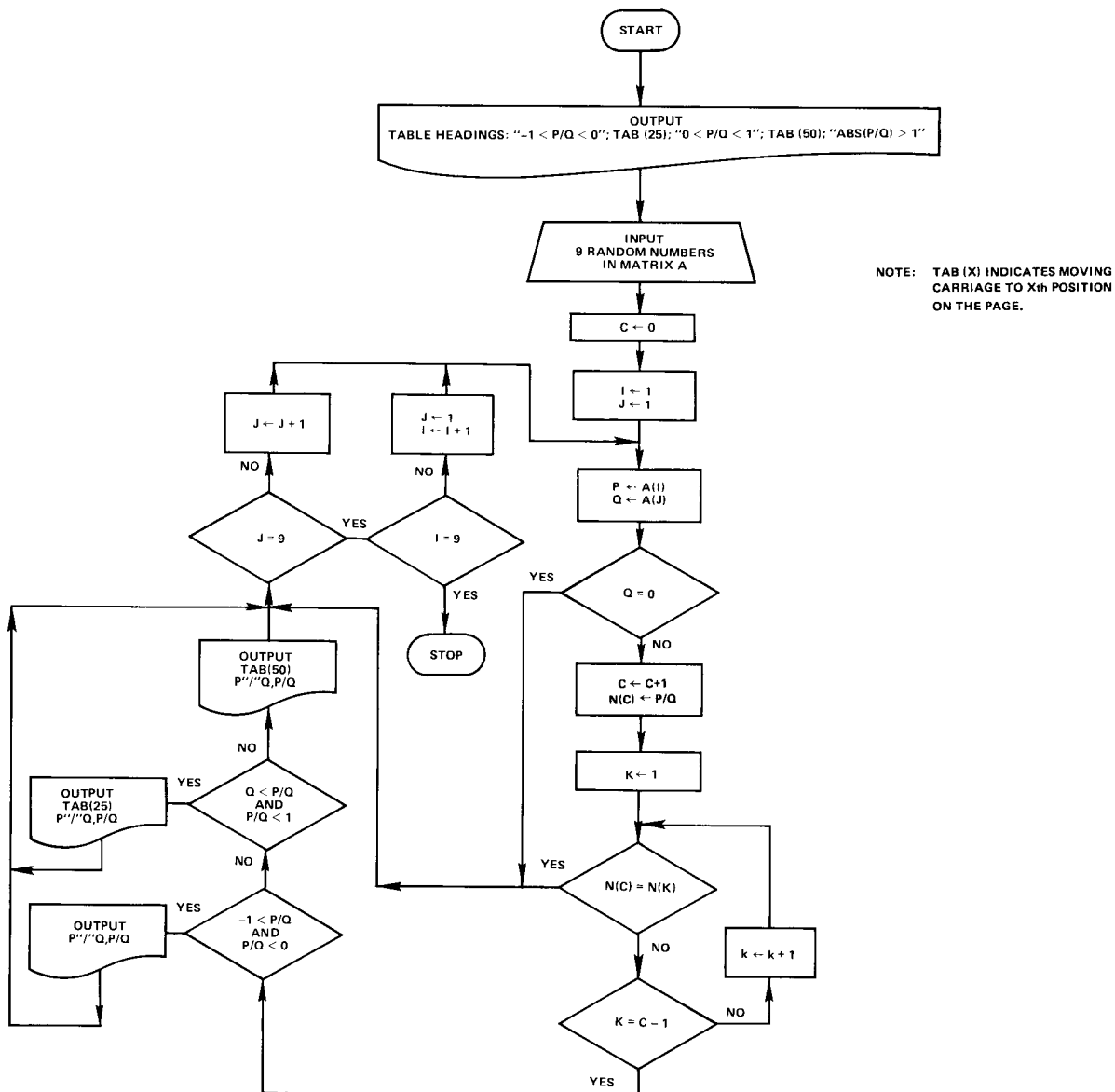
RUN

1	-7	/-7	1
2	-7	/-6	1.16667
3	-7	/-5	1.4
4	-7	/-4	1.75
5	-7	/-3	2.33333
6	-7	/-2	3.5
7	-7	/-1	7
8	-7	/ 1	-7
9	-7	/ 2	-3.5
10	-7	/ 3	-2.33333
11	-7	/ 4	-1.75
12	-7	/ 5	-1.4
13	-7	/ 6	-1.16667
14	-1	/ 1	-1
15	-6	/-7	.857143
16	-6	/-5	1.2
17	-3	/-2	1.5
18	-2	/-1	2
19	-3	/-1	3
20	-6	/-1	6

EXERCISE 9 – Rational Numbers Formed from a Random Set of Integers

This problem is a more complicated case of Exercise 8. It illustrates that the magnitude of the rational number formed is determined by the relationship of numerator to denominator and the sign of each. Also, the idea of a decimal approximation to a rational number is introduced.

Micro Flow Chart



Example Program

Exercise 9.

```

10 REM-- A PROGRAM THAT SEPARATES RATIONAL NUMBERS FORMED INTO
20 REM THREE DIFFERENT CATAGORIES.
30 PRINT "-1<P/Q<0";TAB(25);"0<P/Q<1";TAB(50);"ABS(P/Q)>1"
40 DIM N(100)
50 FOR I=1 TO 9
60 READ A(I)
70 NEXT I
80 DATA -16,3,45,0,-24,-6,7,-19,38
90 LET C=0
100 FOR I=1 TO 9
110 FOR J=1 TO 9
120 LET P=A(I)
130 LET Q=A(J)
140 IF Q=0 THEN 270
150 LET C=C+1
160 LET N(C)=P/Q
170 FOR K=1 TO C-1
180 IF N(C)=N(K) THEN 270
190 NEXT K
200 IF -1<P/Q AND P/Q<0 THEN 240
210 IF 0<P/Q AND P/Q<1 THEN 260
220 PRINT TAB(50);P;"/";Q;P/Q
230 GOTO 270
240 PRINT P;"/";Q;P/Q
250 GOTO 270
260 PRINT TAB(25);P;"/";Q;P/Q
270 NEXT J
280 NEXT I
290 END

```

RUN

$-1 < P/Q < 0$			$0 < P/Q < 1$			$ABS(P/Q) > 1$		
						-16 / -16	1	
						-16 / 3	-5.33333	
-16	/ 45	-.355556						
			-16	/ -24	.666667			
						-16 / -6	2.66667	
						-16 / 7	-2.28571	
			-16	/ -19	.842105			
-16	/ 38	-.421053						
3	/ -16	-.1875						
			3	/ 45	6.66667E-02			
3	/ -24	-.125						
3	/ -6	-.5						
			3	/ 7	.428571			
3	/ -19	-.157895						
			3	/ 38	7.89474E-02			
						45 / -16	-2.8125	
						45 / 3	15	
						45 / -24	-1.875	
						45 / -6	-7.5	
						45 / 7	6.42857	
						45 / -19	-2.36842	
						45 / 38	1.18421	
						0 / -16	0	
						-24 / -16	1.5	
						-24 / 3	-8	
-24	/ 45	-.533333						
						-24 / -6	4	
						-24 / 7	-3.42857	
						-24 / -19	1.26316	
-24	/ 38	-.631579						
			-6	/ -16	.375			
						-6 / 3	-2	
-6	/ 45	-.133333						
			-6	/ -24	.25			
-6	/ 7	-.857143						
			-6	/ -19	.315789			
7	/ -16	-.4375						
			7	/ 45	.155556			
						7 / 3	2.33333	
7	/ -24	-.291667						
						7 / -6	-1.16667	
7	/ -19	-.368421						
			7	/ 38	.184211			
						-19 / -16	1.1875	
-19	/ 45	-.422222				-19 / 3	-6.33333	
			-19	/ -24	.791667			
						-19 / -6	3.16667	
						-19 / 7	-2.71429	
						38 / -16	-2.375	
						38 / 3	12.6667	
			38	/ 45	.844444			
						38 / -24	-1.58333	
						38 / 7	5.42857	

DONE

IRRATIONAL NUMBERS

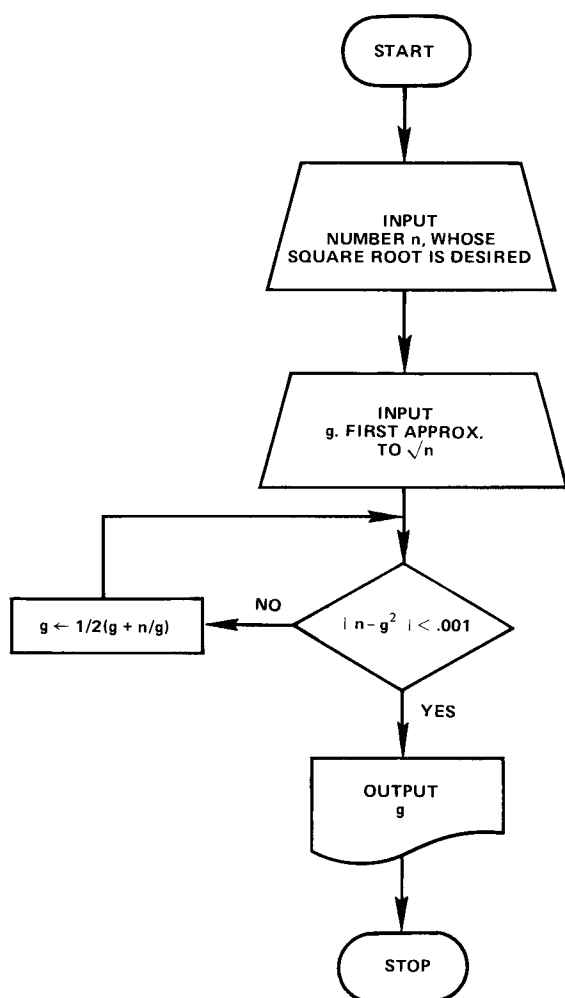
EXERCISE 10 – A Rational Approximation of \sqrt{n} , n a rational number > 0

Newton's method (averaging) of finding the square root of a number has become the most popular square root algorithm used in algebra texts. Your students will appreciate the opportunity to use the computer to perform the tedious and cumbersome arithmetic operations involved.

In Parts (b), (c), and (d), it is important that your students understand the development of the equations defining the algorithms used in each program. The exercises themselves are subordinate to this understanding. Part (d) will be most valuable if the students are expected to research the algorithm given for the solution.

Micro Flow Chart

Part (a).



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Example Program

Exercise 10(a).

```
10 REM--A PROGRAM TO FIND THE SQUARE ROOT OF A NUMBER BY
15 REM--NEWTON'S METHOD
20 INPUT N,G
30 IF ABS(N-G*2)<.001 THEN 60
40 LET G=(G+N/G)/2
45 PRINT G
50 GOTO 30
60 PRINT "APPROXIMATE SQUARE ROOT OF";N;"IS";G
70 END
```

RUN

```
?5,3
2.33333
2.2381
2.23607
APPROXIMATE SQUARE ROOT OF 5    IS 2.23607
```

DONE
RUN

```
?16,5
4.1
4.00122
4
APPROXIMATE SQUARE ROOT OF 16   IS 4
```

DONE
RUN

```
?16,3
4.16667
4.00333
4.
APPROXIMATE SQUARE ROOT OF 16   IS 4.
```

DONE
RUN

```
?72,5
9.7
8.56134
8.48562
8.48528
APPROXIMATE SQUARE ROOT OF 72   IS 8.48528
```

DONE

Example Program

Exercise 10(b).

```
10 REM--APROGRAM TO FIND THE CUBE ROOT OF A NUMBER USING
20 REM--NEWTON'S METHOD.
30 INPUT N,G
40 IF ABS(N-G^3)<.001 THEN 70
50 LET G=(G+N/G^2)/2
55 PRINT G
60 GOTO 40
70 PRINT "APPROXIMATE CUBE ROOT OF";N;"IS";G
80 END
```

RUN

```
?27,5
3.04
2.98079
3.00979
2.99515
3.00244
2.99879
3.00061
2.9997
3.00015
2.99992
3.00004
2.99998
APPROXIMATE CUBE ROOT OF 27 IS 2.99998
```

DONE
RUN

```
?27,3
APPROXIMATE CUBE ROOT OF 27 IS 3
```

DONE
RUN

```
?-8,4
1.75
-.431123
-21.7364
-10.8766
-5.47214
-2.86965
-1.92056
-2.04471
-1.9791
-2.01078
-1.99469
-2.00267
-1.99867
-2.00067
-1.99967
-2.00017
-1.99992
APPROXIMATE CUBE ROOT OF -8 IS -1.99992
```

DONE

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Example Program

Exercise 10(d).

```
5  REM--A PROGRAM TO COMPUTE THE CUBE ROOT OF A NUMBER.
10  INPUT N,G
20  LET H=1/3*(2*G+N/G^2)
25  PRINT H;
30  IF ABS(N-H^3)<.001 THEN 60
40  LET G=H
50  GOTO 20
60  PRINT "THE CUBE ROOT OF ";N;"IS";H
70  END
```

RUN

```
?16,2
2.66667      2.52778      2.51987      THE CUBE ROOT OF  16   IS 2.51987
```

DONE
RUN

```
?27,5
3.69333      3.12201      3.00471      3.00001      THE CUBE ROOT OF  27
IS 3.00001
```

DONE
RUN

```
?63,7
5.09524      4.20572      3.99105      3.97909      3.97906
THE CUBE ROOT OF  63   IS 3.97906
```

DONE
RUN

```
?-8,4
2.5           1.24         -.907638      -3.8421      -2.74205      -2.1827
-2.01487      -2.00011      -2           THE CUBE ROOT OF -8   IS -2
```

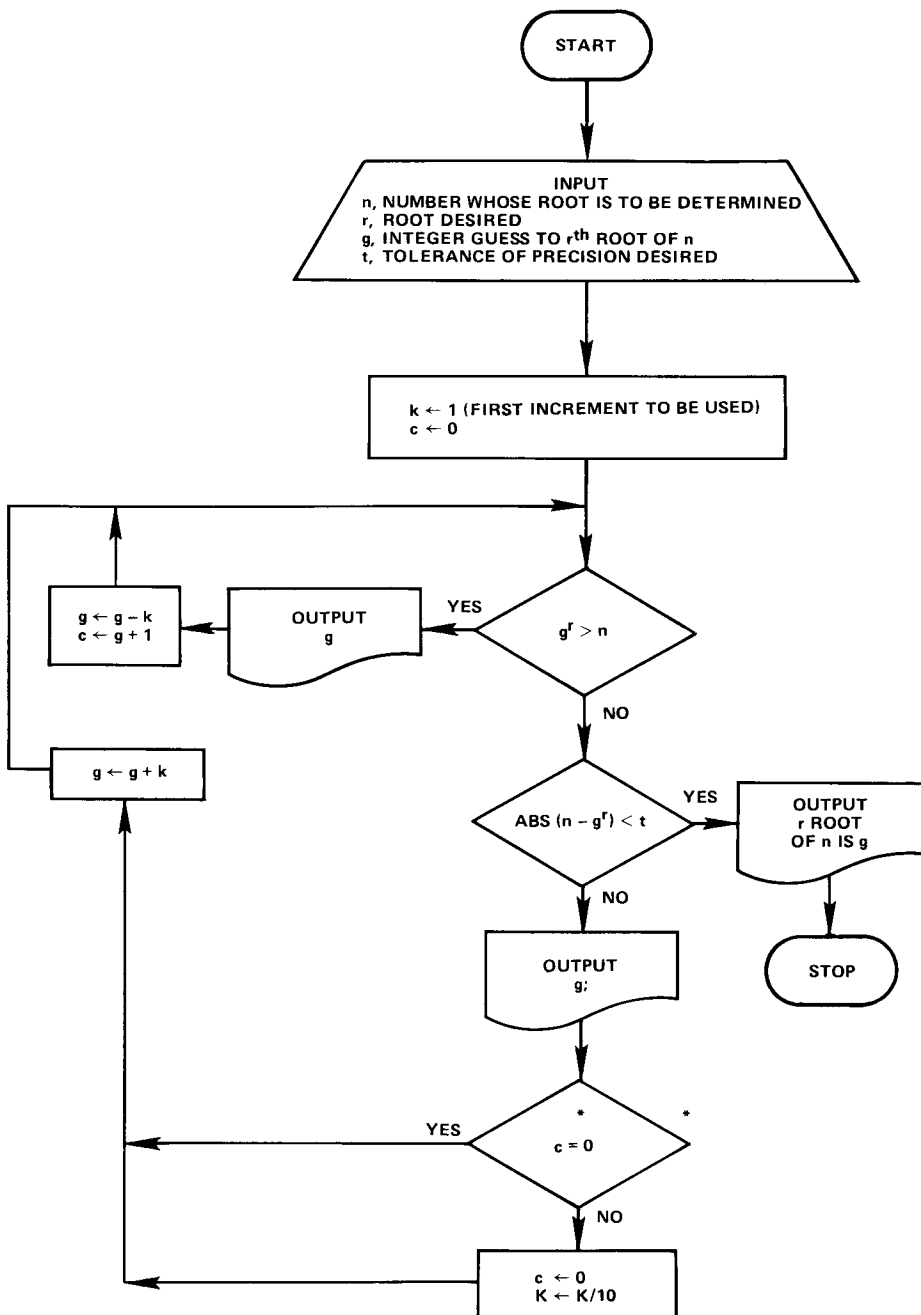
DONE

EXERCISE 11 – Incrementing Method for Finding $\sqrt[n]{n}$, n Rational

This problem is valuable in that it can serve as an introduction to the concept of a limit and the various root search methods used to find roots of polynomial equations.

The variable c is kept and incremented if $g > \sqrt[n]{n}$. Later it is tested for 0 to determine whether the increment should be changed.

Micro Flow Chart



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Example Program

Exercise 11.

```

5  REM--A PROGRAM FOR FINDING THE RTH. ROOT OF A NUMBER BY AN
6  REM--ITERATIVE PROCESS.
10  INPUT N,R,G,T
20  LET K=1
30  LET C=0
40  IF G*R>N THEN 120
50  IF ABS(N-G*R)<T THEN 160
60  PRINT G;
70  IF C=0 THEN 100
80  LET C=0
90  LET K=K/10
100 LET G=G+K
110 GOTO 40
120 PRINT G;
130 LET G=G-K
140 LET C=C+1
150 GOTO 40
160 PRINT R;"TH. ROOT OF ";N;"IS";G
170 END

```

RUN

```

?16,3,5,.001
5      4      3      2      2.1      2.2      2.3      2.4
2.5      2.6      2.5      2.51      2.52      2.51
2.511      2.512      2.513      2.514      2.515      2.516
2.517      2.518      2.519      2.52      2.519      2.5191
2.5192      2.5193      2.5194      2.5195      2.5196      2.5197
3      TH. ROOT OF 16      IS 2.5198

```

DONE

RUN

```

?81,4,2,.001
2      4      TH. ROOT OF 81      IS 3

```

DONE

RUN

```

?.0234,2,.01,.00001
.01      1.01      9.99999E-03      .11      .21      .11
.12      .13      .14      .15      .16      .15
.151      .152      .153      .152      .1521      .1522
.1523      .1524      .1525      .1526      .1527      .1528
.1529      .153      .1529      .15291      .15292      .15293
2      TH. ROOT OF .0234      IS .15294

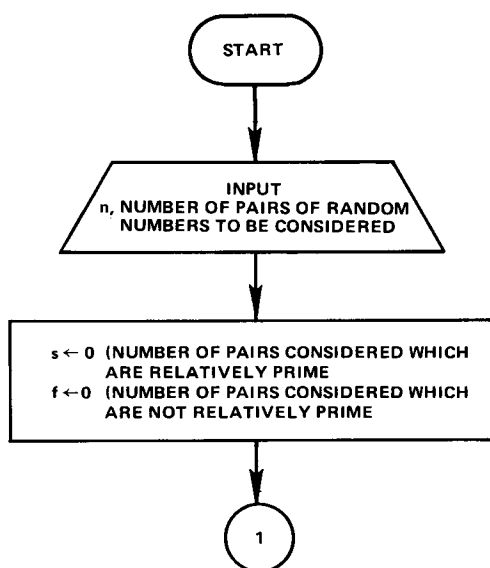
```

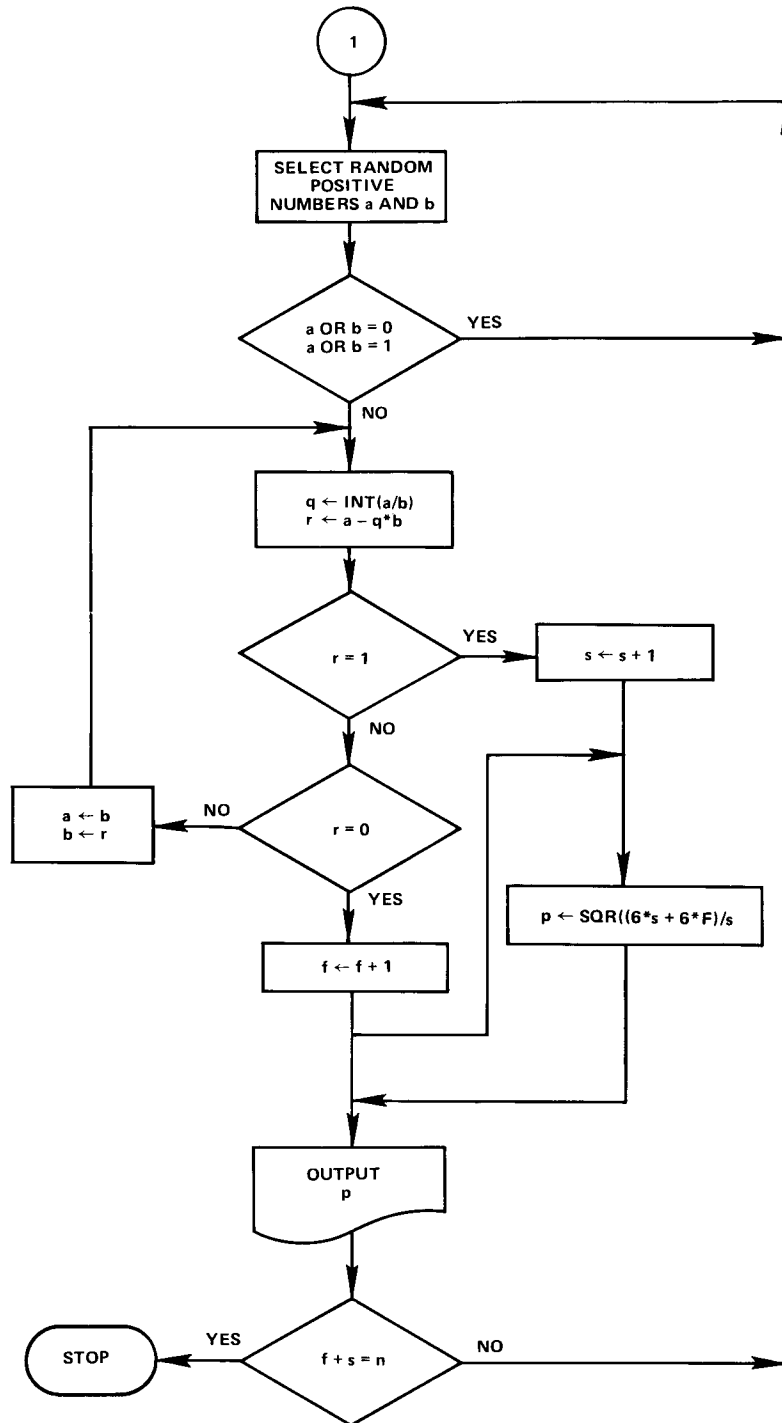
DONE

EXERCISE 12 – Computing π by a Probability Method

The random number–probability approach to computing π , instead of a circle-related approach, is presented here since it ties in well with the students' study of numbers. Encourage your students to have the computer print the approximation to π after the selection of each pair of random numbers. This gives the student the opportunity to see that the successive approximations gradually approach the actual value of π . Different runs of the program will result in approximations sometimes approaching π as a lower bound, and other times approaching π as an upper bound. Consequently, the problem is a good introduction to the concept of a limit.

Micro Flow Chart





Example Program

Exercise 12.

```

10 REM--A PROGRAM TO COMPUTE PI.
20 LET S=0
30 LET F=0
40 INPUT N
50 FOR X=1 TO N
60 LET A=INT(RND(0)*1000)
70 LET B=INT(RND(0)*1000)
80 IF A=0 OR B=0 THEN 200
90 IF A=1 OR B=1 THEN 200
100 LET Q=INT(A/B)
110 LET R=A-Q*B
120 IF R=1 THEN 170
130 IF R=0 THEN 220
140 LET A=B
150 LET B=R
160 GOTO 100
170 LET S=S+1
180 LET P=SQR((6*S+6*F)/S)
190 PRINT P;
200 NEXT X
210 GOTO 60
220 LET F=F+1
230 IF S=0 THEN 200
240 GOTO 180
250 END

```

RUN

```

?20000
2.44949      3.4641      3      2.82843      3.16228      3.4641
3.24037      3.09839      3      2.9277      3.0706      3.20714
3.33809      3.24037      3.3541      3.4641      3.3665      3.4641
3.55903      3.65148      3.74166      3.63318      3.54196      3.4641
3.53553      3.60555      3.53009      3.59487      3.52542      3.4641
3.40955      3.36067      3.41278      3.3665      3.32455      3.28634
3.25137      3.21926      3.26134      3.30289      3.34392      3.31006
3.27872      3.24962      3.22252      3.19722      3.23179      3.26599
3.24037      3.27327      3.24834      3.2249      3.20282      3.2329
3.21131      3.1909      3.17157      3.19926      3.1803      3.16228
3.18852      3.21455      3.19628      3.17888      3.16228      3.18651
3.21056      3.19374      3.21714      3.20061      3.18479      3.20714
3.22933      3.21334      3.23498      3.25648      3.24037      3.2249
3.24551      3.26599      3.25042      3.23544      3.25511      3.27466
3.2596      3.24509      3.2311      3.24962      3.23583      3.25396
3.27198      3.25813      3.24473      3.26213      3.24893      3.23616
3.25297      3.2697      3.28634      3.30289      3.28959      3.30584
3.322      3.33809      3.32455      3.31142      3.29869      3.28634
3.30152      3.31663      3.33167      3.3191      3.30688      3.29502
3.28348      3.27226      3.28634      3.27525      3.2891      3.27815
3.26749      3.28096      3.29438      3.30774      3.32106      3.33431
3.32317      3.31231      3.30173      3.3145      3.30404      3.29383
3.28387      3.27414      3.26464      3.25537      3.26731      3.27921
3.26991      3.26081      3.25192      3.26343      3.2749      3.28634
3.27731      3.28859      3.29983      3.29079      3.30188      3.29293
3.28416      3.27558      3.26717      3.25894      3.2695      3.28003
3.27178      3.26369      3.274      3.26599      3.25813      3.25042
3.24286      3.25279      3.24529      3.23793      3.24767      3.24037
3.25      3.24276      3.25228      3.26177      3.27124      3.2639
3.25669      3.26599      3.27525      3.26803      3.26093      3.25396
3.24711      3.25606      3.265      3.25813      3.26697      3.26015
3.25345      3.24685      3.25548      3.26408      3.25747      3.25097
3.25943      3.25297      3.24662      3.24037      3.23422      3.22817
3.22221      3.21634      3.22437      3.23238      3.22649      3.23443

```

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3.22858	3.22282	3.21714	3.2249	3.21926	3.2137
3.20822	3.21581	3.21037	3.21789	3.21248	3.20714
3.20187	3.19668	3.19156	3.18651	3.18153	3.18872
3.18377	3.17888	3.17406	3.1693	3.1646	3.17157
3.17851	3.18544	3.19236	3.19926	3.20615	3.20128
3.19647	3.19172	3.19847	3.20521	3.20045	3.19574
3.1911	3.19771	3.20431	3.19965	3.19505	3.1905
3.18601	3.19246	3.19889	3.19438	3.20077	3.19628
3.19185	3.18748	3.19374	3.2	3.20624	3.20182
3.20802	3.20362	3.19926	3.19496	3.19071	3.18651
3.18236	3.18835	3.18422	3.19018	3.19612	3.19197
3.18787	3.19374	3.18966	3.19549	3.19142	3.19722
3.19317	3.18916	3.19489	3.20061	3.19659	3.19261
3.18868	3.18479	3.18093	3.17712	3.17335	3.16962
3.16593	3.16228	3.15866	3.15508	3.16048	3.15691
3.15338	3.14989	3.1552	3.15172	3.157	3.16228
3.15878	3.15532	3.16054	3.16575	3.17095	3.16746
3.164	3.16914	3.16569	3.17081	3.16737	3.16397
3.16059	3.16564	3.17068	3.16729	3.1723	3.16893
3.16559	3.16228	3.16721	3.16392	3.16882	3.17373
3.17862	3.17529	3.17199	3.16873	3.16549	3.16228
3.16707	3.17185	3.16863	3.17338	3.17813	3.1749
3.1717	3.16853	3.16539	3.17006	3.16692	3.17157
3.16844	3.16535	3.16228	3.15924	3.15622	3.15323
3.15776	3.15478	3.15182	3.14889	3.14598	3.1431
3.14024	3.13741	3.14181	3.13898	3.14336	3.14774
3.1449	3.14926	3.14643	3.15076	3.14794	3.14514
3.14943	3.14664	3.15091	3.14813	3.15238	3.1496
3.15383	3.15106	3.15527	3.15948	3.1567	3.15394
3.15811	3.15537	3.15951	3.16366	3.1609	3.15817
3.15546	3.15955	3.16364	3.16773	3.1718	3.16906
3.17312	3.17038	3.16766	3.16496	3.16228	3.16628
3.16361	3.16759	3.17157	3.16889	3.16623	3.16359
3.16097	3.16489	3.16228	3.15968	3.15711	3.15456
3.15202	3.1495	3.147	3.14452	3.14833	3.15214
3.14965	3.15345	3.15096	3.1485	3.14605	3.14362
3.14736	3.14494	3.14253	3.14014	3.14384	3.14754
3.15123	3.14882	3.14643	3.15009	3.14771	3.15135
3.14897	3.15261	3.15023	3.14787	3.15148	3.15508
3.15868	3.16228	3.15989	3.15752	3.16109	3.15872
3.16228	3.15992	3.15757	3.1611	3.16463	3.16228
3.16579	3.1693	3.16694	3.1646	3.16809	3.16575
3.16922	3.16689	3.16458	3.16802	3.16571	3.16914
3.16684	3.16455	3.16228	3.16002	3.15777	3.16115
3.16453	3.16228	3.16004	3.15782	3.15561	3.15895

STOP